Quadratics Assignment

Ques 1

a) $y = -3x^2 + 2x + 1$

$= -3\left[x^2 - \frac{2}{3}x - \frac{1}{3}\right]$

$= -3\left[x^2 + 2 \times x \times \left(\frac{-1}{3}\right) + \left(\frac{-1}{3}\right)^2 - \left(\frac{-1}{3}\right)^2 - \frac{1}{3}\right]$

$= -3\left[\left(x - \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{1}{3}\right]$

$= -3\left[\left(x - \frac{1}{3}\right)^2 - \frac{4}{9}\right]$

$\boxed{y = -3\left(x - \frac{1}{3}\right)^2 + \frac{4}{3}}$ vertex form.

b) axis of symmetry $x = \frac{1}{3}$.

direction of opening $= -3 < 0$, downward opening.

x intercept = by putting $y = 0$

$0 = -3\left(x - \frac{1}{3}\right)^2 + \frac{4}{3}$

$+\frac{4}{3} = \cancel{+}3\left(x - \frac{1}{3}\right)^2$

$$\left(x - \frac{1}{3}\right)^2 = \frac{4}{9}$$

$$x - \frac{1}{3} = \pm \frac{2}{3}$$

$$x = \frac{1}{3} \pm \frac{2}{3}$$

$$x = \frac{1}{3} + \frac{2}{3} \text{ or } \frac{1}{3} - \frac{2}{3}$$

$$= \frac{\cancel{3}}{\cancel{3}} = 1 \text{ or } -\frac{1}{3}$$

y intercept by putting $x = 0$.

$$y = -3\left(-\frac{1}{3}\right)^2 + \frac{4}{3}$$

$$= +3 + \frac{1}{3} + \frac{4}{3}$$

$$= -\frac{1}{3} + \frac{4}{3}$$

$$= 1.$$

maximum value $= \frac{4}{3}$.

Ques 2. given $h = -3$, $k = 2$

$$|a| = \frac{1}{2}$$

opens up, $\therefore a = +\frac{1}{2}$

$$= \frac{1}{2}(x-(-3))^2 + 2$$

$$= \frac{1}{2}\left[(x+3)^2\right] + 2$$

$$= \frac{1}{2}\left[x^2 + 9 + 6x\right] + 2$$

$$= \frac{1}{2}x^2 + 3x + \frac{9}{2} + 2$$

$$= \frac{1}{2}x^2 + 3x + \frac{13}{4}.$$

Ques 3. Let the first number be x.
Let the second number be y.

$\therefore$, according to question

$$x + 3y = 18 \quad \text{——} (1)$$

$\therefore$, product $\cancel{P} = x \times y$.

$$P = xy.$$

from ①, $y = \frac{18-x}{3}$

$\therefore P = x\left(\frac{18-x}{3}\right)$

$= -\frac{x^2}{3} + 6x$

for quadratic form

$P = -\frac{1}{3}[x^2 - 18x]$

$= -\frac{1}{3}[x^2 + 2x(-9) + (-9)^2 - (9)^2]$

$= -\frac{1}{3}[(x-9)^2 - 81]$

$= -\frac{1}{3}(x-9)^2 + 27$

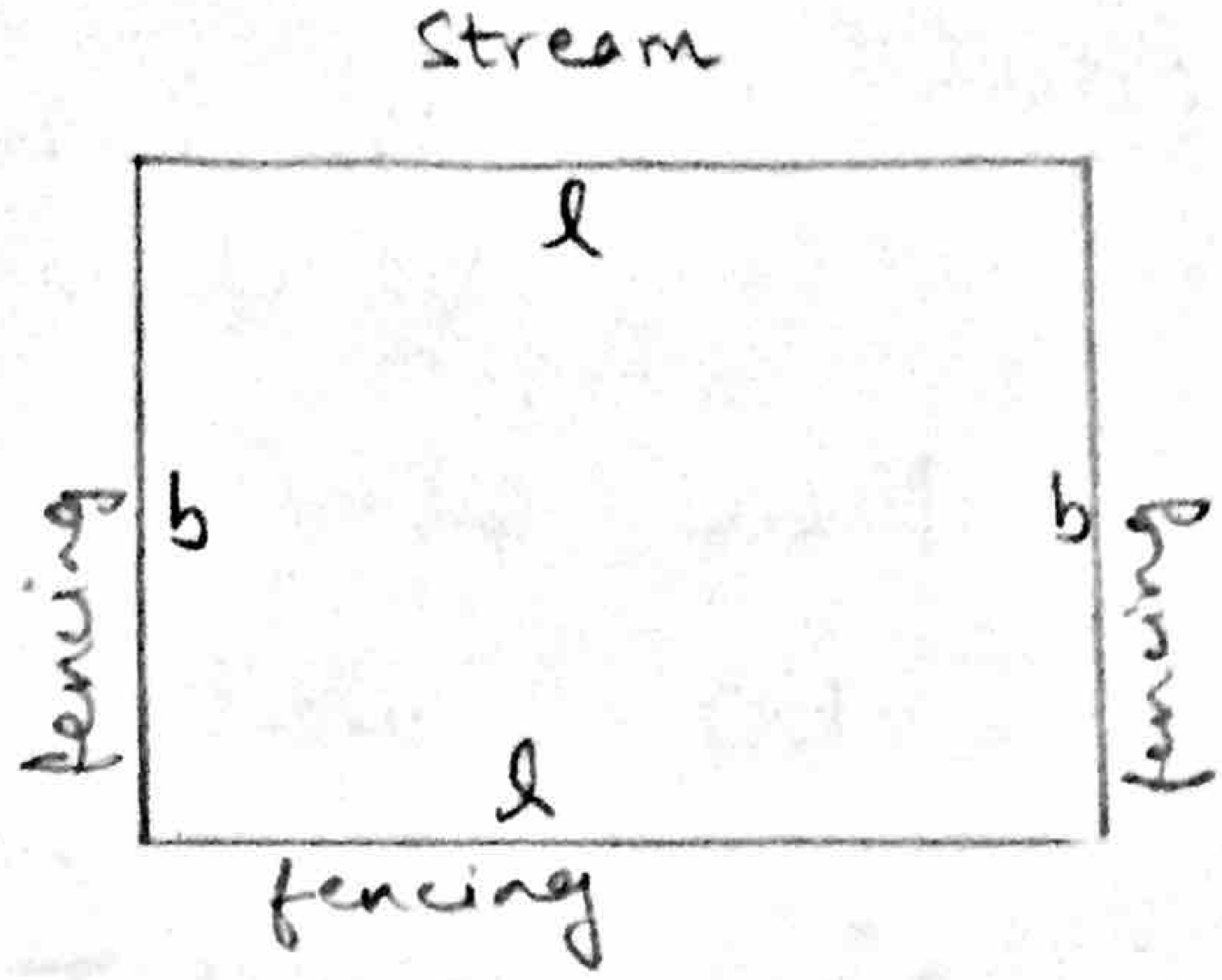
As $a = -\frac{1}{3} < 0$, so, P will be maximum at $x = 9$ and $P = 27$.

$\therefore y = \frac{P}{x} = \frac{27}{9} = 3$

first number = 9

second number = 3.

Ques 4.



According to question

$$2b + l = 600 \quad \text{---(i)}$$

area $= lb$

$$A = b(600 - 2b)$$
$$= 600b - 2b^2$$
$$= 2b^2 + 600b$$
$$= -2[b^2 - 300b]$$
$$= -2[b^2 - 2 \times b \times 150 + (150)^2 - (150)^2]$$
$$= -2[(b-150)^2 - 22500]$$
$$= -2[(b-150)^2] + 45000$$

$-2 < 0$, Area is maximum $= 45000$ at $b = 150$.

$$l = \frac{4500\cancel{0}}{15\cancel{0}} = 300$$

Hence dimensions are,

$l = 300\,m$
$b = 150\,m$

Ques 5. Let the increase in Admission cost be (dollar) $\$x$

then, admission cost $= (\$ 8+x)$

No. of visitors $= (2000-100x)$

$\therefore$, revenue $= (8+x)(2000-100x)$

$= 16000 - 800x + 2000x - 100x^2$

$= -100x^2 + 1200x + 16000$

$= -100[x^2 - 12x - 160]$

$= -100[x^2 + 2\times x\times(-6) + (-6)^2 - (6)^2 - 160]$

$= -100[(x-6)^2 - 36 - 160]$

$= -100[(x-6)^2 - 196]$

$= -100(x-6)^2 + 19600$

$\therefore$, $x = 6$, $(6, 19600)$

$h = 6$, $k = 19600$

a) Equation of Revenue $= R(x) = -100(x-6)^2 + 19600$

b) Coordinate of the maximum point of the function $= (6, 19600)$.

(C) Admission cost for maximum Revenue $= 8 + 6 = \$14$

(d) Number of visitors for maximum Revenue $= 2000 - 100 \times 6$

$= 1400.$